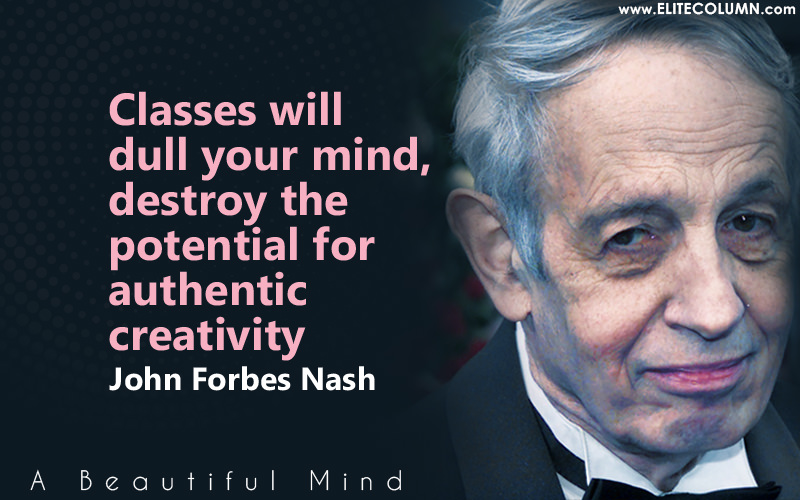
John Nash Society Website

***“Where rational behavior meets irrational behavior”***

******

***RPI: Rational numbers (R) meets Irrational numbers (Pi)***

Contents

[Page: Who was John Nash 4](#_Toc208265146)

[John Nash’s Contributions to the World 4](#_Toc208265147)

[Personal Life 4](#_Toc208265148)

[Impact on the World 4](#_Toc208265149)

[Page: What we’re about 5](#_Toc208265150)

[Mission 5](#_Toc208265151)

[Vision 5](#_Toc208265152)

[Strategic Objectives 5](#_Toc208265153)

[Page: Leadership: 6](#_Toc208265154)

[Page: Club Activities 7](#_Toc208265155)

[Learning & Discussion 7](#_Toc208265156)

[Applied Game Nights 7](#_Toc208265157)

[Competitions 7](#_Toc208265158)

[Projects 7](#_Toc208265159)

[Community & Outreach 7](#_Toc208265160)

[ AAtmosphere 8](#_Toc208265161)

[Page: Intro to Game Theory 9](#_Toc208265162)

[Lecture 1: The World of Game Theory 9](#_Toc208265163)

[Lecture 2: The Nature of the Game 16](#_Toc208265164)

[Lecture 3: The Real Life Chessboard Sequential Games 20](#_Toc208265165)

[Lecture 4: Life's Little Games -The 2x2 Classic Games 32](#_Toc208265166)

[Lecture 5: Guessing Right -Simultaneous Move Games 38](#_Toc208265167)

[Lecture 6: Practical Applications of Game Theory 38](#_Toc208265168)

[Lecture 7: A Random Walk -Dealing with Chance Events 38](#_Toc208265169)

[Lecture 8: Pure Competition -Constant Sum Games 38](#_Toc208265170)

[Lecture 9: Mixed Strategies and Nonzero-sum Games 38](#_Toc208265171)

[Lecture 10: Threats, Promises, and Commitments 38](#_Toc208265172)

[Lecture 11: Credibility, Deterrence, and Compellence 38](#_Toc208265173)

[Lecture 12: Incomplete and Imperfect Information 38](#_Toc208265174)

[Lecture 13: Whom can you trust? Signaling and Screening 38](#_Toc208265175)

[Lecture 14: Encouraging Productivity Incentive Schemes 38](#_Toc208265176)

[Lecture 15: The Persistence of Memory Repeated Games 38](#_Toc208265177)

[Lecture 16: Does this stuff really work? 38](#_Toc208265178)

[Lecture 17: The Tragedy of the Commons 38](#_Toc208265179)

[Lecture 18: Game Theory in Motion: Evolutionary Game Theory 38](#_Toc208265180)

[Lecture 19: Game Theory and Economics: Oligopolies 38](#_Toc208265181)

[Lecture 20: Voting -Determining the Will of the People 38](#_Toc208265182)

[Lecture 21: Auctions and the Winner's Curse 38](#_Toc208265183)

[Lecture 22: Bargaining and Cooperative Games 38](#_Toc208265184)

[Lecture 23: Game Theory and Business Coopetition 38](#_Toc208265185)

[Lecture 24: All the World's a Game 38](#_Toc208265186)

# Page: Who was John Nash

**John Nash** (1928 – 2015) was an American mathematician whose work fundamentally shaped **game theory, economics, and mathematics**. He’s most famous for the concept of the **Nash Equilibrium**, which describes a stable outcome in a game where no player can improve their situation by unilaterally changing strategy.

## John Nash’s Contributions to the World

* **Game Theory**:
  + Introduced the **Nash Equilibrium** (1950), a cornerstone of modern economics, political science, and AI multi-agent systems.
  + Showed that in any finite game, at least one equilibrium exists (sometimes involving mixed strategies).
* **Mathematics**:
  + Contributions in **differential geometry** and **partial differential equations (PDEs)**.
  + His work on nonlinear PDEs earned him the **Abel Prize (2015)**.
* **Economics**:
  + Awarded the **1994 Nobel Prize in Economic Sciences**, shared with Reinhard Selten and John Harsanyi, for pioneering analysis of equilibria in non-cooperative games.

## Personal Life

* Studied at Princeton and MIT, producing groundbreaking work in his 20s.
* Struggled with **schizophrenia** for decades, retreating from public life.
* Made a remarkable recovery and returned to academic work later in life.

## Impact on the World

* Nash Equilibrium is used in:
  + Economics (market competition, auctions)
  + International relations (arms races, treaties)
  + Biology (evolutionary stable strategies)
  + Computer science & AI (multi-agent systems, auctions, negotiation, reinforcement learning)

## Page: What we’re about

## Mission

To foster a community of curious minds who explore the principles of game theory, mathematics, and strategic thinking inspired by John Nash, while promoting collaboration, innovation, and resilience in the face of challenges.

## Vision

To be a leading student hub that cultivates future thinkers, innovators, and problem-solvers who apply the lessons of John Nash’s work and life to create positive impact in academia, industry, and society.

## Strategic Objectives

**Academic Engagement**

* Host seminars, workshops, and talks on game theory, economics, decision science, and mathematics.
* Partner with professors and guest speakers to connect Nash’s theories to real-world applications (AI, business strategy, political science).
* Organize peer study groups and competitions like “Game Theory Challenges.”

**Interdisciplinary Projects**

* Collaborate with business, economics, political science, and computer science clubs on case studies and simulations.
* Develop projects or research exploring Nash equilibrium in everyday life (e.g., environmental issues, negotiation, technology adoption).

**Innovation & Problem-Solving**

* Create an annual **“Nash Innovation Challenge”** where students design solutions to global or campus problems using game-theoretic approaches.
* Promote mathematical modeling competitions.

**Mental Health & Resilience Awareness**

* Dedicate initiatives to mental health awareness, inspired by Nash’s life journey.
* Partner with counseling services to host events about balancing intellectual pursuit with personal well-being.

**Community & Outreach in the Capital Region**

* Mentor younger students in math and economics.
* Build outreach programs with local schools to inspire interest in problem-solving and critical thinking.
* Organize film screenings (e.g., *A Beautiful Mind*) with follow-up discussions on lessons from Nash’s life.

# Page: Leadership:

* President
* Vice President
* Events Coordinator
* Treasurer
* Outreach/PR
* Faculty Advisors
* Project Manager
* Marketing

Positions elected annually in the Spring to take effect in the Fall.

# Page: Club Activities

**Meetings:** Weekly or biweekly, usually 1–2 hours.

## Learning & Discussion

* Short lectures or workshops on core topics: Nash equilibrium, repeated games, evolutionary dynamics, mechanism design.
* Reading groups for key texts (*The Evolution of Cooperation* by Axelrod, *The Strategy of Conflict* by Schelling, etc.).
* Guest talks from professors or grad students who use game theory in research.

## Applied Game Nights

* Playing and analyzing strategy-heavy games (e.g., Diplomacy, Risk, Settlers of Catan, Coup, Avalon, Poker).
* Running simulations of famous problems (Prisoner’s Dilemma tournaments, Public Goods games, Auction games).
* Coding sessions for algorithmic game theory or agent-based simulations.

## Competitions

* Internal tournaments: Iterated Prisoner’s Dilemma bots coded by members.
* Inter-club collaborations (e.g., with debate, econ, or chess clubs).
* Participation in online competitions related to strategy or AI.

## Projects

Semester

* + Modeling real-world issues: voting systems, market design, climate cooperation.
  + Writing short papers/blog posts on strategy insights.
  + Building simple apps to simulate and visualize games.

Long Term Projects

* + Create a Monopoly game that can be used to simulate real world
    - Negotiations
    - Valuations
    - Accounting

## Community & Outreach

* Social events with strategy games (casual + academic).
* Outreach to local high schools with workshops on game theory basics.
* Collaboration with economics, political science, and AI clubs.

## AAtmosphere

* A balance between **academic rigor** (for those interested in research or graduate study) and **fun/gameplay** (to keep it engaging).
* Mix of students who like math proofs, programming simulations, and those who just love strategy games.
* A culture of playful competition but cooperative spirit (meta-game theory in action).

# Page: Intro to Game Theory

// list of lectures on this page, each lecture gets its own page

## Lecture 1: The World of Game Theory

***If you know the enemy and know yourself, you need not fear the result of a hundred battles.***

***If you know yourself but not the enemy, for every victory gained you will also suffer a defeat.***

***If you know neither the enemy nor yourself, you will succumb in every battle.***

***Game Theory is the study of strategic interaction among rational players***

***"Sun Tzu - Art of War"***

* Interactions are "Data driven (AI) decision making (Human)"
* **Games Theory is less than 100 year old**

**Sample Game: Push or not to push? That is the question**

* **Backstory**
  + The goal of the game is for an individual player to keep as much of the $100 as they can
  + Assuming each player will act rationally, a player will choose the option that maximizes the amount of money they keep
* **Rules**
  + 51 players are given $100 and a button
  + Pushing the button limits a players losses
    - If a player pushes their button, they lose $1 for every other player that pushed their button
    - If a player doesn't push their button, they lose $2 for every other player that pushed their button
  + No cooperation. Players have to make their decision without knowing what other players have decided
* **Expected Payoffs**
  + How much money a player keeps depends on the other 50 players
  + Below is a Game Tree, also known as a Decision Tree

A diagram of a company

AI-generated content may be incorrect.

* Note:
  + The Game Tree in the example above is incomplete. It is missing the probability of a decision being made
  + You will later learn how to build a complete tree.
  + For now, assume every decision is equally likely

* **Common Knowledge**
  + Everyone knows the rules and understand how to maximize their Expected Payoffs
* **Play the game**
  + On the count of three….one…..two….THREE
* **Results**
  + Did you or didn't you push the button?
    - What was your thought process?
  + Did others push their button?
    - What was their thought process?

* **Analyzing the Results**
  + Results
    - Between 30% and 70% (average 50%) push the button
  + Just looking at the payoffs, the best outcome is everyone NOT to push the button
    - No one loses any money
    - Question is, did everyone else agree?
  + This result goes against common sense.
    - Everyone loses some part of the $100, unless you are the only one that pushed their button.
  + Possible reason everyone pushes their button
    - "Someone else is going to push their button, so I limit my losses by pushing my button"
    - "If I don't push the button and someone else does, I end up with my worst expected payoff"
  + This is the **Prisoner's Dilemma**: the most important game in Game Theory

* **Congratulation! You play your first game.** 
  + Was it the outcome you expected?

* **Now let's do a thought experiment**
  + You're the President of the United States
    - You're at war and playing this game with another nuclear nation
    - **… would you push the button?**
  + ***"It's all fun and games until someone gets hurt"***

**Parts of a Game**

* Rational Players
  + Make decisions that maximize their expected payoffs
* Common Knowledge
  + Everyone knows the rules
  + Everyone knows the same information about the game at the same time
* Expected Payoffs
  + What players expect to win (what players value)
  + Different players in the same game may have different payoffs
  + Payoffs are not always measured in $
* Strategy
  + Set of choices a player will make and the expected payoffs associated with those choices
  + **Dominant Strategy**: The set of choices that maximizes expected payoffs
* Strategy profile
  + A collection of strategies
  + Strategies interact with other strategies

**Game Tree (aka Extensive form of a game)**

* Visual map
  + All the possible decisions a player could make in a game
  + The probabilities that decision will occur
  + The expected payoff for a sequence of decisions
* Root node
  + Beginning of the tree
  + First decision to be made
* Decision nodes
  + Each subsequent decisions that need to be made as the game is played
* Branches
  + Available choices based on decision node
  + Probabilities of these choices occurring
* Leaf nodes
  + Payoffs at the end after all decisions in a branch have been made
* Reading **left** to **right**, players determine different ways (**strategies**) the game can be played
  + Strategy
    - A unique path, driven by a series of decisions, that leads to a payoff as defined in the rules
  + Strategy Profile:
    - The collection of strategies.
    - The Game Tree is your strategy profile

A diagram of a decision

AI-generated content may be incorrect.

* Reading **right** to **left**, a player calculates the expected payoffs for each strategy
  + An **Expected Payoff** is the product of a payoff and its probabilities for each decision
  + At each decision (where branches converge) a player determines the incoming branch with the highest expected payoff
    - That branch becomes part of the players Dominant Strategy
  + **Dominant Strategy** the path with the highest Expected Payoff

\*\*\* How to create a Decision Tree in PowerPoint\*\*\*

Example: Game Tree for "Push the Button Game"

* Create a bulleted hierarchy like below
  + Push the button?
    - If you pushed button
      * You lose $1 for every player that pushed their buttons
      * Every player that didn't push loses $2
    - If you don't push the button
      * You lose $2 for every player that pushed their button

* Open PowerPoint > Illustrations > SmartArt >
  + A screenshot of a computer

    AI-generated content may be incorrect.
* Hierarchy > Horizontal Hierarchy>
  + Add an extra box at the end of each branch to hold the payoffs

A screenshot of a computer

AI-generated content may be incorrect.

* Paste the hierarchy into text box
  + A screenshot of a computer

    AI-generated content may be incorrect.

**John von Neumann**

* *The Theory of Games and Economic Behavior(1944)*
  + Started out as strategy for playing poker
  + Wanted to study Economics quantitatively as a science
  + Developed the Minimax Theorem (1928)

**Thomas Schelling**

* *Strategy of Conflict (1960)*
  + "How do you make a credible threat?"

**Reihard Selten**

* Ex-post rationality
  + Look backwards to see how earlier situations could have been done better rather than look forward and examine the current situation in its own right

**Game #2: The Dollar Auction Game**

* **Rules**
  + A $1 bill will be auctioned
  + Winner pays bid and gets the $1 bill
  + 2nd place still has to pay their bid, even though they didn't win the $1
* Play the game
  + The bidding starts at $0
* Results
  + The winning bid was $ ?!?
  + Not what you expected? Were these Rational Players?
* Analysis
  + Why would a Rational Player pay more than $1 for $1
  + Reason: **No one wants to be in second place**
    - The Player in second place keeps bidding with the Player in first place to avoid paying for nothing
    - The last two players keep bidding until one gives up
    - How high they bid depend on group psychology and competitive dynamics (and how much money they had on them)
* *“Dollar Auction”* game was invented by economist Martin Shubik
  + It was an experiential exercise illustrating how rational decisions can lead to escalating irrational losses.
  + When played with Wall Street executives, $100 sold for $467
  + It first appeared in Shubik’s 1971 paper *“The Dollar Auction Game: A Paradox in Noncooperative Behavior and Escalation”*
* It is the classic ***"War of Attrition"***
  + Both sides pour resources into the battle trying to get the other side to quit
  + Eventually, one side runs out of resources or decides the cost of continuing is too much
  + Everyone loses in the end

**Episode 2: Nature of the Game**

## Lecture 2: The Nature of the Game

**Types of Game**

* Sequential Games (Dynamic Games)
  + Game develops overtime
  + Players take turns
  + e.g. Monopoly
* Simultaneous Games (Static Games)
  + Game happens all at once
  + Players make play at the same time
  + e.g. Sealed Bid Auction

**Length of Games**

* Finite
  + Game must end
  + Finite number of choices
  + Finite number of players
  + Order is all that matters when ranking payoffs
* Infinite
  + Game doesn't end
  + We will fill in the blanks later

**Types of strategy**

* Pure Strategy
  + Doesn't involve chance
  + e.g. Chess
* Mixed Strategy
  + Involves some amount of chance
    - Note: Arbitrary choices are not the same random choices
  + e.g. Monopoly (rolling dice)

**Payoffs**

* For every game, a player makes choices that maximize their expected payoffs
* Represent the actual preferences of each player
  + Can be measured in different units
    - Payoffs for different players can't be directly compared to the payoffs of other players
  + In general, the bigger the payoff the better
* Payoff types
  + Ordinal
    - **Definition**: Ordinal numbers express **position or rank**, but not magnitude.
    - **Examples**: 1st, 2nd, 3rd...
    - **Interpretation**:
      * "I prefer A > B > C" → just a ranking, no info on how *much* better A is than B.
    - **Operations**: You can compare ("better than", "worse than"), but not meaningfully subtract or add.
    - **Use case**: Ordinal utility in economics assumes only **ranked preferences**, without assuming how much more one is preferred over another.
    - For finite games, all that matters is how a player ranks the payoffs

* Cardinal
  + Players rank their preferences on an Interval scale
  + **Definition**: Cardinal numbers express **quantity**.
  + **Examples**: 1, 2, 3, 10, 100...
  + **Interpretation**:
    - "I have **3 apples**" → tells you *how many*.
    - "Utility of Option A is **80**, Option B is **40**" → Option A is *twice as desirable*.
  + **Operations**: You can add, subtract, multiply, compare differences, etc.
  + **Use case**: Cardinal utility in economics assumes **intensity** of preference (e.g., how much more you prefer A to B).
  + For games that are not finite, you have to be careful what units you measure payoffs
* Expected
  + Payoff is multiplied by the probability of the choice(s) happening

**Game #3 - Ultimatum Game**

* Rules
  + $10 on table
  + Player 1 proposes a split of the $10 with Player 2
  + Player 2 can accept or reject
* Play the game
  + Player 1 proposes keeping $9.99 and giving Player 2 $0.01
* Analysis
  + Should Player 2 accept the offer?
  + If Player 2 is rational, they should accept this split, since $0.01 is better than nothing
  + However, put yourself in Player 2's place.
    - If Player 2 is sensitive to fairness, they may want the payoff to be split more evenly
    - Does Player 1 have any obligation to do so?
  + Game is NOT symmetric. No obligation to be fair

**Game #4: Do you feel lucky?**

* Rules
  + Player is offered $1
    - Choice 1:
      * Take $1 and the game ends
    - Choice 2: Flip a coin (0.5 probability)
      * Heads player wins $10
      * Tails player loses the $1
      * Expected payoff is $5 (average of $10 and $0)

* A diagram of a graph

  AI-generated content may be incorrect.
* Play the Game
  + A rational player should choose to flip a coin
    - Expected payoff for flipping the coin is $5
    - Expected payoff for walking away is $1
    - $5 > $1
    - Flipping the coin maximizes players Expected Payoff

* Play the game again, but add 6 zero's
  + A diagram of a diagram

    AI-generated content may be incorrect.
  + Did you choose to flip the coin?
    - Most players would take $1,000,000 and not risk the coin flip
  + **Game Theory says you shouldn't care**
    - Expected value of flipping is $5,000,000
    - Expected payoff for walking away is $1,000,000
    - $5,000,000 > $1,000,000
  + Irrational thinking led players to an outcome that didn't maximize their expected payoffs
  + Game Theory solves this dilemma by using Utils to normalize payoffs

**Utility Function**

* A **util** represents **one unit of satisfaction**.
* It’s **not a real or standardized unit** like kilograms or dollars.
* It’s a **conceptual tool** to compare preferences.

**Common Knowledge**

* "I know that you know that I know that you know that I know…."

## Lecture 3: The Real Life Chessboard Sequential Games

***Thinking Strategically (1991)***

* Dixit and Nalebuff

**Sequential Games rules**

* Finite (the games end)
* Deterministic (no chance involved)
* Non-cooperative (players don't know what other players are going to do)
* Sequential (players take turns)
* Perfect information
  + Each player knows the rules of the game
  + Each player knows the history of a game before they make a move
* Dynamic

**Compete or Cooperate?**

* Cooperative game
  + Binding agreements
* Non-cooperative game
  + Binding agreements not possible

**Strategy and Strategy Profiles**

* Strategy
  + How a player will respond to a specific series of moves by another player
  + A path between the Root node (start of the game) and the Leaf node (Payoff) is a separate strategy
* Strategy Profile
  + A collection of Strategies
  + A playbook for how a player will play the game under any circumstance regardless of what other players do

**Dominant Strategy**

* Best strategy a player can play from the player's strategy profile (maximizes expected payoffs)
  + Players should choose their Dominant Strategy regardless of what other players do

**Rollback to decide what strategy maximize payoffs**

* *"To look forward, you must reason backwards"*
* **Kuhn's Theorem**
  + The rollback procedure will give you the right answer for any finite, deterministic, sequential, noncooperative game of perfect information
  + The best way to play any game

**Nash equilibrium (Best strategy)**

* Each player calculates their Strategy Profile
* Each player choose their Dominant Strategy from their Strategy Profile
  + The strategy that maximizes their Expected Payoffs
* Nash equilibrium states
  + No player will gain from unilaterally changing their strategy away from their Dominant Strategy
* **Subgame perfection**
  + Multiple Nash equilibriums can exist in a game
    - Nash equilibriums alone can't narrow down the best choice among multiple Nash equilibriums
  + Start at any point in the tree
    - If strategy works at all possible starting points, then it’s a Nash equilibrium

**Winner of any game: player who doesn't make a mistake (always play your dominant strategy)**

**Case Study: Game of Free Trade**

**Goal of the Game**

* Airbus needs to decide whether to build a factory
* EEC and US need to decide whether or not to implement Protective Legislation (PL).
  + Protective Legislation in this case means that only airplane manufacturers from that market can sell.
  + Creates a monopoly for the airplane manufacturer

**Common Knowledge**

* Competition exists between two airplane manufactures (Airbus and Boeing) in two economic markets (EEC and US)

**Players**

* Boeing
* Airbus
* US Market
* EU Market (EEC: European Economic Community)
* Nature (Optional player)
  + Accounts for probability
  + We will omit probability from this analysis

**Payoffs**

* Airbus startup costs
  + $1b to build new factory
* Closed market (PL)
  + Monopoly over a market
    - $900m for the company
      * No competition
      * Gets all the customers
    - $900m for the market
      * Increases in tax revenue from supporting air travel infrastructure
      * Multiplier affect
* Competition in a market (No PL)
  + $300m for company
    - Available customers split between Boeing and Airbus
    - Ticket prices reduced due to competition
  + $300m for market
    - Reduces tax revenue due to ticket price reduction

$700m for competition bonus

* Competition good for the market
* Increase in tax revenue for other companies related to air travel
* Reduce prices increase customers
* Multiplier affect

**All the games here are non-cooperative**

* No negotiating among players

**Games**

1. PL in both markets and Airbus builds
2. PL in both markets and Airbus doesn't build
3. PL in EEC and Airbus builds
4. Free trade and Airbus builds

**Order of play:**

1. EEC decides if it's going to pass PL
2. If EEC passes PL, US decides if it's going to retaliate
3. Airbus decides whether to build a factory or not

**Game Matrix**

* Matrix form of the game tree showing expected payoffs
* Use these matrices to fill out the Game Tree

**PL in both market , Airbus doesn't build**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Airbus** | **EEC** | **Boeing** | **US** |
| **US Market** |  |  | 900 | 900 |
| **EEC Market** |  |  | 900 | 900 |
| **Dev Costs** | - | -- | -- | -- |
| **Comp Bonus** | -- | -- | -- |  |
| **Payoff** | 0 | 0 | 1800 | 1800 |

**PL in both markets, Airbus builds**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Airbus** | **EEC** | **Boeing** | **US** |
| **US Market** | 0 | 0 | 900 | 900 |
| **EEC Market** | 900 | 900 | 0 | 0 |
| **Dev Costs** | -1000 | -- | -- | -- |
| **Comp Bonus** | -- | -- | -- | -- |
| **Payoffs** | -100 | 900 | 900 | 900 |

**PL in EEC only , Airbus builds**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Airbus** | **EEC** | **Boeing** | **US** |
| **US Market** | 300 | 300 | 300 | 300 |
| **EEC Market** | 900 | 900 | -- | -- |
| **Dev Costs** | -1000 | -- | -- | -- |
| **Comp Bonus** | -- | -- | -- | 700 |
| **Payoff** | 200 | 1200 | 300 | 1000 |

**Both Markets open , Airbus builds**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Airbus** | **EEC** | **Boeing** | **US** |
| **US Market** | 300 | 300 | 300 | 300 |
| **EEC Market** | 300 | 300 | 300 | 300 |
| **Dev Costs** | -1000 | -- | -- | -- |
| **Comp Bonus** | -- | 700 | -- | 700 |
| **Payoff** | -400 | 1300 | 600 | 1300 |

**Game 1 Game Tree: EEC > US > Airbus**

Note: Boeing's Dominant strategy is to maximize expected payoff in any market its allowed, so it doesn't need to be added to the Game Tree

A diagram of a plane

AI-generated content may be incorrect.

**Determine Dominant Strategy:**

* Work backwards (right to left)
  + When you reach a decision node, compare the payoff to the other decision that led to this decision node
  + Whatever payoff is higher becomes part of the Dominant Strategy
  + Whatever path reaches the Root Node is the Dominant strategy
* Below is the Game Tree narrative from above to demonstrate the process of working backwards.
  + Payoffs are color coded to match the color of the player
  + Bold is the Dominant Strategy
  + Italics represents neutral (could choose either way

* *EEC passes PL*
  + ***US retaliates with PL***
    - If Airbus builds, they lose $100m
      * US makes $900m
        + EU makes $900
    - **If Airbus doesn't build, they breakeven $0**
      * **US makes $1800**
        + **EU makes $0**
  + *US doesn't pass PL*
    - **If Airbus builds, they make $200**
    - If Airbus doesn't build, they breakeven $0

**If EEC passes PL and US retaliates, it's better for Airbus not to build**

* *EEC doesn't pass PL*
  + *US doesn't retaliate*
    - If Airbus builds, they make $200m
      * US makes $1000
        + EU makes $1200
    - **If Airbus doesn't build, they breakeven $0**
      * **US makes $1800**
        + **EU makes $0**

**If EEC doesn't pass PL, and the US doesn't pass PL, it's still better for Airbus not to build**

**Results of the Game**

* Final Decisions
  + EEC is neutral
    - It makes no money either way
  + US is neutral
    - It makes $1800m either way
    - But if EEC passes PL, US retaliates with PL
  + Airbus doesn't build in either case
* Expected payoffs
  + US = $1,800
  + EEC = 0
  + Boeing = $1,800
  + Airbus = 0

**Does order matter?**

* Let Airbus decide whether to build before US decides if it will pass PL

**Game 2:**

Order of play: EEC > Airbus > US

**Game 2 Game Tree**

A diagram of a plane

AI-generated content may be incorrect.

* **EEC passes PL**
  + **If Airbus builds, they make $200m**
    - **US doesn't pass PL**
      * **US makes $1000m**
      * **EU makes $1200m**
  + If Airbus doesn't build, they breakeven $0
    - US neutral
      * US makes $1800 either way
      * EU makes $0
    - **If EEC passes PL and US retaliates, it's better for Airbus not to build**
* EEC doesn't pass PL
  + US doesn't retaliate
    - If Airbus builds, they lose $400m
      * US makes $1000
        + EU makes $1200
    - If Airbus doesn't build, they breakeven $0
      * US makes $1800
        + EU makes $0

**Play the game**

* Decisions
  + EEC passes PL
  + Airbus builds
  + US doesn't pass PL
* Expected Payoffs
  + EEC = $1,200
  + US = $1,000
  + Boeing = $300
  + Airbus = $200

**PL in EEC only , Airbus builds**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Airbus** | **EEC** | **Boeing** | **US** |
| **US Market** | 300 | 300 | 300 | 300 |
| **EEC Market** | 900 | 900 | -- | -- |
| **Dev Costs** | -1000 | -- | -- | -- |
| **Comp Bonus** | -- | -- | -- | 700 |
| **Payoff** | 200 | 1200 | 300 | 1000 |

**Order DOES matter in this case**

* EEC and Airbus does better when Airbus moves before US

* **Note that changing the order of turns doesn't change the original payoff's**
  + Payoffs depend of the what events occurred, not the order in which they happened

**Game 3: Let Airbus decide whether to build before it knows about PL**

Order of play: Airbus> EEC > US

**Game 3 Game Matrix: PL in EEC only , Airbus builds**

A diagram of a plane

AI-generated content may be incorrect.

**Outcome of the Game:**

* Should Airbus build?
  + Airbus builds
    - EEC passes PL(1300 > 1200)
      * US passes PL
        + **900 900 -100**
      * US doesn't pass PL (1000 > 900)
        + 1200 1000 200
    - EEC doesn't pass PL (1300 > 900)
      * 1300 1300 -400
  + Airbus doesn't build (-100 > -400)
    - 0 1800 0

**Outcome**

* Decisions
  + Airbus doesn't build
  + EEC is neutral
  + US passes PL
* Expected payoffs
  + US = $1,800
  + EEC = 0
  + Boeing = $1,800
  + Airbus = 0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Airbus** | **EEC** | **Boeing** | **US** |
| **US Market** |  |  | 900 | 900 |
| **EEC Market** |  |  | 900 | 900 |
| **Dev Costs** | - | -- | -- | -- |
| **Comp Bonus** | -- | -- | -- |  |
| **Payoff** | 0 | 0 | 1800 | 1800 |

* Back to original scenario, even with different order of turns
* It's better for Airbus to go second
  + EEC makes more money with Airbus in competitive market
  + Better outcome is when they work together

**Order matters, but going first is not a guaranteed better outcome**

* **First mover advantage**
  + Commitment to course of action
    - Payoffs remain the same no matter what order
  + Doesn't always guarantee better outcome
* **Follower's advantage**
  + Flexibility of response
  + Learn from First Mover's mistakes

When US goes before Airbus, then EEC is neutral and Airbus is on its own

**Airbus's delicate balance.**

* It's best outcome
  + Wait for EEC to pass PL
  + Decide whether to build before US decides whether to retaliate with PL
* **Follower's advantage** with respect to EEC **First Mover advantage** with respect to the US

**Rollback Equilibrium**

* Nash Equilibrium
  + No one gets a better payoff by unilaterally changing their strategy
* Determining Nash Equilibrium
  + Each player selects their Dominant Strategy from their strategy profile
    - Does any player have regrets?
    - Would any player change their strategy once they knew what another players strategy was.
  + If no, then it is a Nash Equilibrium
* Subgame Perfection
  + The strategy is a Nash Equilibrium if
    - You can start the game at any point in the game
    - And it is still a Nash Equilibrium

## Lecture 4: Life's Little Games -The 2x2 Classic Games

**Episode 4: Life's Little Games - 2x2 Classic Games - Static Games**

**Simultaneous Games (Static)**

* Players make their decision at same time
* Players don't know what other players decisions are when they make theirs

**Properties of an Atomic Game (2X2 game)**

* Smallest game that can be played
  + Two players
  + Two choices for each player
  + 2 by 2 Game matrix
* Ordinal payoffs
  + How the player values the payoff
  + Order matters, exact size difference doesn't
* Finite
  + Game must end (only one move)

**Atomic Game examples**

* Game #1: Coordination game
* Game #2: Battle of the sexes
* Game #3: Chicken
* Game #4: Prisoner's dilemma

**Game #1 Coordination Game**

**Dinner date: Deciding what to wear**

* Similar preferences
  + Both agree that formal is better than casual
  + Both agree that matching is the better than not matching

**Game Matrix definitions**

* Two pairs of numbers in each cell separated by a comma represent the Payoffs for each player and the combination of choices
  + First number and color is the row player
  + Second number and color is the column player

**Game Matrix Dinner Date**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | P2 |  |
|  |  | Formal | Casual |
| P1 | Formal | 2,2 | 0,0 |
|  | Casual | 0,0 | 1,1 |

* Payoffs
  + Both match: Both get 2
    - Best outcome
  + Both Casual: Both get 1
    - Acceptable outcome
  + Mixed Formal/Casual: Both get 0
    - Worse outcome
* Common Knowledge
  + Both know each other's preferences

**Rational Players will be honest about preferences**

* A player is Irrational when they don't choose their dominant strategy
* "Even though this is not my preferred choice, I choose my partners choice because I don't want to start a fight"

**Focal point (Schelling)**

* Obvious choice to all players in the game with common knowledge
* Example
  + 6 Teams of 2; all strangers
  + $100 given and dropped off in NYC
  + Goal: Meet up at same place and same time
  + Result
    - Place: Time Square or Empire State Bldg
    - Time: Noon

**Nash equilibrium(s) in Coordination Game**

* "No one gets a better payoff by unilaterally changing their strategy"
  + Both formal and both casual are Nash equilibriums
  + Neither will move from matching to not matching
    - Moving from Formal to Mixed loses 2 points
    - Moving from Casual to Mixed loses 1 point
    - Moving from Formal to Casual loses 1 point
* Schelling Point
  + Given the Common knowledge is they both prefer Formal, they will probably both dress formal

**Game #2: Battle of the Sexes**

* Different preferences
  + P1 prefers Casual,
  + P2 prefers Formal
  + No utility in not matching
  + No Schelling Point

**Game Matrix: Battle of the Sexes**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | P2 |  |
|  |  | Formal | Casual |
| P1 | Formal | 1,2 | 0,0 |
|  | Casual | 0,0 | 2,1 |

* No clear Dominant Strategy
  + No strategy leaves both satisfied
    - Not Pareto Optimal
  + Solution: Change the game
  + **Turn the simultaneous game into a sequential game**
    - Call and leaving message saying what you're going to wear
    - First mover advantage

**Game #3: Chicken ("A Rebel Without a Cause")**

* Two cars drive at each other
* Whoever swerves first is a chicken
* Payoff's
  + Whoever swerves first is the chicken
    - Player who swerves is labeled a chicken (1)
    - Player who didn't swerve gets bragging rights (3)
    - Not good to be labeled a chicken
  + If both swerve
    - Both are labeled chickens, but at least they are alive (2 each)
    - Nash equilibrium
  + If neither swerve
    - Both are legends, but it doesn't matter because they are both dead (0 each)

**Game Matrix: Chicken**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Driver 2 |  |
|  |  | Don't swerve | Swerve |
| Driver 1 | Don't swerve | 0,0 | 3,1 |
|  | Swerve | 1,3 | 2,2 |

**Game #4: Prisoner's Dilemma**

* **Most important game in Game Theory**
  + The **Prisoner's Dilemma** was invented by **Merrill Flood** and **Melvin Dresher** at the RAND Corporation in **1950**. However, the formal **game-theoretic structure and name "Prisoner’s Dilemma"** were introduced shortly after by **Albert W. Tucker**, a Princeton mathematician.

* Two criminals committed a crime together, are captured, and are held in separate cells for questioning
  + Each has two choices: Rat or Don't Rat
    - If neither rats, they both go free
    - If one rats and the other doesn't
      * The one who ratted gets 1 year (but is labeled a rat)
      * The other gets 5 years
    - If both rat, they each get three years (and both are labeled rats)

**Game Matrix: Prisoner's Dilemma**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | P2 |  |
|  |  | Rat | Don’t Rat |
| P1 | Rat | -3,-3 | -5,-1 |
|  | Don't Rat | -1,-5 | -0,-0 |

* Play the game
  + No matter what the other prisoner does, it's better to rat (in the short term)
    - First mover advantage
    - Even though the best payoff is for both not to rat (in the short term AND the long term)
  + Not Pareto Optimal
    - Both have an incentive to rat
      * Change from Cooperation(don't rat) to Betrayal(rat)
    - Neither is maximizing their expected payoffs

**Why is ratting actually bad?**

* Ratting breaks the code of criminals, which has a worse expected payoff
  + Short term: You get a shive in prison
  + Long term: You get whacked when you get out

## Lecture 5: Guessing Right -Simultaneous Move Games

## Lecture 6: Practical Applications of Game Theory

## Lecture 7: A Random Walk -Dealing with Chance Events

## Lecture 8: Pure Competition -Constant Sum Games

## Lecture 9: Mixed Strategies and Nonzero-sum Games

## Lecture 10: Threats, Promises, and Commitments

## Lecture 11: Credibility, Deterrence, and Compellence

## Lecture 12: Incomplete and Imperfect Information

## Lecture 13: Whom can you trust? Signaling and Screening

## Lecture 14: Encouraging Productivity Incentive Schemes

## Lecture 15: The Persistence of Memory Repeated Games

## Lecture 16: Does this stuff really work?

## Lecture 17: The Tragedy of the Commons

## Lecture 18: Game Theory in Motion: Evolutionary Game Theory

## Lecture 19: Game Theory and Economics: Oligopolies

## Lecture 20: Voting -Determining the Will of the People

## Lecture 21: Auctions and the Winner's Curse

## Lecture 22: Bargaining and Cooperative Games

## Lecture 23: Game Theory and Business Coopetition

## Lecture 24: All the World's a Game